UNIT 1 WS SOLUTIONS

1.1) $x = A \cos \omega t$

$A = 7\,cm \qquad \omega = 6\pi\ s^{-1}$

a) $f = \frac{\omega}{2\pi} = 3\,Hz$

b) $T = \frac{2\pi}{\omega} = \frac{1}{f} = \frac{1}{3}\,s \quad (= 0.333\,s)$

c) $A = 7\,cm$

d) $x = 0, \quad t > 0$

$\Rightarrow \cos \omega t = 0$

$\omega t = \frac{\pi}{2}$

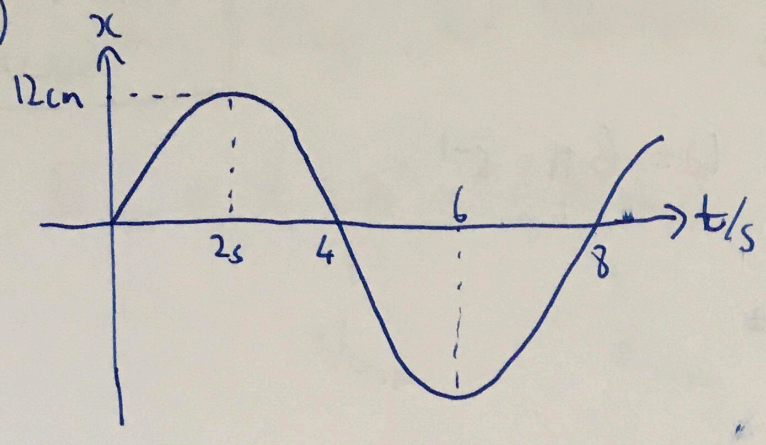
$t = \frac{\pi}{2\omega} = \frac{1}{12}\,s \quad (= 0.0833\,s)$

$\frac{dx}{dt} = v = -A\omega \sin \omega t, \quad \omega = \frac{\pi}{2} \Rightarrow \sin \omega t = 1$

$\Rightarrow v = -A\omega < 0$

$\Rightarrow$ -ve X-Dirn

1.2)




$x = A \sin \omega t$

$A = 12\,cm \quad \omega = \frac{\pi}{4}\,s^{-1}$

a) $0s \to 2s$ : $A = 12\,cm$

b) $2s \to 4s$ : $A = 12\,cm$

c) $0s \to 1s$ : $x(t=1s) = A \sin \frac{\pi}{4} = A \cdot \frac{1}{\sqrt{2}} = 8.49\,cm.$

d) $1s \to 2s$ : ~~$12\,cm = A \sin \frac{\pi}{4}$~~ =

$A\left(1 - \sin \frac{\pi}{4}\right) = 3.51\,cm$

1.3) $m = 1.5\,kg$ $k = 500\,N/m$ $v_{max} = 0.7\,m/s$

a) $E_{TOT} = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2 = 0.368\,J$

$\frac{1}{2}kA^2$ = $U_{max}$ ; $\frac{1}{2}mv_{max}^2$ = $K_{max}$

b) $A = \sqrt{\frac{2E_{TOT}}{k}} = \sqrt{\frac{mv_{max}^2}{k}} = 3.83\,cm$

1.4) $A = 0.058\,m$ $v_{max} = 2.2\,m/s$ $k = 1.8 \times 10^3\,N/m$

a) $U_{max} = K_{max}\ (= E_{TOT})$

$\Rightarrow kA^2 = mv_{max}^2$

$m = \frac{kA^2}{v_{max}^2} = 1.25\,kg$

b) $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{v_{max}^2}{A^2}} = \frac{v_{max}}{A}$

$f = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\frac{v_{max}}{A} = 6.04\,Hz$

c) $T = \frac{1}{f} = \frac{2\pi A}{v_{max}} = 0.166\,s$