PA1140 – Waves and Quanta

Unit 4 – Core Exercises Feedback

Exercise 4.1. The neutron, when isolated from an atomic nucleus, decays into a proton, an electron, and an anti-neutrino as follows:

$$ {}^{1}_{0}\mathrm{n} \rightarrow {}^{1}_{1}\mathrm{H} + {}^{0}_{-1}\mathrm{e} + {}^{0}_{0}\bar{v}. $$

The thermal energy of a neutron is of the order of kT, where k is the Boltzmann constant.

a) For a thermal neutron at 25 °C, find its energy in both joules and electron volts.

b) What is the speed of this thermal neutron?

c) A beam of monoenergetic thermal neutrons is produced at 25 °C with an intensity I. After travelling 1350 km, the beam has an intensity $I/2$. Using this information, estimate the half-life of the neutron. Express your answer in minutes.

(a) Energy of neutron $E = kT = 1.38\times10^{-23}\times298 = \boxed{4.11\times10^{-21}\ \mathrm{J} = 0.0257\ \mathrm{eV}}$

(b) Velocity of neutron $v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\times4.11\times10^{-21}}{1.67\times10^{-27}}} = \boxed{2.22\times10^{3}\ \mathrm{m\ s^{-1}}}$

(c) Half life = Travel time = $t_{1/2} = \frac{1350}{2.22} = 608\ \mathrm{s} = \boxed{10.1\ \mathrm{min}}$

Exercise 4.2. The rubidium isotope ^{87}Rb is a β emitter with a half-life of 4.9×10^{10} y that decays into ^{87}Sr. It is used to determine the age of rocks and fossils. Rocks containing the fossils of early animals contain a ratio of ^{87}Sr to ^{87}Rb of 0.0100. Assuming that there was no ^{87}Sr present when the rocks were formed, calculate the age of these fossils.

Number of half-lives, n: $N = N_0\left(\frac{1}{2}\right)^n \Rightarrow n = \frac{\ln(\frac{N}{N_0})}{\ln(1/2)} \approx \frac{\ln(0.99)}{\ln\left(\frac{1}{2}\right)} \approx 0.0145$

Hence age: $t = n \times t_{1/2} = 4.9 \times 10^{10} \times 0.0145 \approx \boxed{700 \text{ Myr}}$

Exercise 4.3. The total energy consumed in the United States in 1 y is approximately 7.0×10^{19} J. How many kilograms of ^{235}U would be needed to provide this amount of energy if we assume that 200 MeV of energy is released by each fissioning uranium nucleus, that all of the uranium atoms undergo fission, and that all of the energy-conversion mechanisms used are 100 percent efficient?

Energy released by each fission $E = 200 \text{ MeV} = 3.2\times10^{-11} \text{ J}$

Thus, number of fissions required to produce annual energy requirement:

$$N = \frac{7\times10^{19}}{3.2\times10^{-11}} \approx 2.2\times10^{30}$$

Requiring a mass of ^{235}U*:*

$$M \approx 2.2\times10^{30}\times235\times1.67\times10^{-27} \approx \boxed{8.6\times10^{5} \text{ kg}}$$

Exercise 4.4. The wavelengths λ of spectral lines produced by the Hydrogen atom are given by the expression

$$\frac{1}{\lambda} = 1.096776 \times 10^{-2} \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right) \text{nm}^{-1}$$

Where n_1, n_2 are integers with $n_1 > n_2$.

$a)$ Calculate the energy in eV required to ionize Hydrogen.

$b)$ Consider the electron transitions creating the Pfund ($n_2 = 5$) series. Find the largest and smallest wavelengths for this series.

$c)$ Calculate the energies associated with these transitions.

(a) Of course, this is just $\boxed{E = 13.6 \text{ eV}} = 2.18 \times 10^{-18}$ J

(b) Smallest wavelength when $n_1 \to \infty$ *i.e.* $\frac{1}{\lambda} = \frac{1.096776 \times 10^{-2}}{5^2} \Rightarrow \boxed{\lambda = 2279 \text{ nm}}$

Largest wavelength when $n_1 = 6$ *i.e.* $\frac{1}{\lambda} = 1.096776 \times 10^{-2} \left(\frac{1}{5^2} - \frac{1}{6^2} \right) \Rightarrow \boxed{\lambda = 7460 \text{ nm}}$

(c) Photon energy $E = \frac{hc}{\lambda} = \frac{1240}{2279} = \boxed{0.544 \text{ eV}}$ *and* $E = \frac{1240}{7460} = \boxed{0.166 \text{ eV}}$

Exercise 4.5. The age of an ancient boat is determined by comparing the radioactive decay of carbon from living wood with that of wood taken from the ancient boat. A sample of 3.00×10^{23} atoms of carbon is removed for investigation from a block of living wood. In living wood one in 10^{12} of the carbon atoms is of the radioactive isotope, which has a decay constant of 3.84×10^{-12} s^{-1}.

a) Calculate the half-life of the wood in years.

b) Find the initial rate of decay of the carbon atoms in the living wood sample in Bq.

c) A sample of 3.00×10^{23} atoms of carbon is removed from a piece of wood taken from the ancient boat. The rate of decay due to the atoms in this sample is 0.65 Bq. Calculate the age of the ancient boat in years.

(a) Half life $$t_{1/2} = \ln(2)\,\tau = \frac{\ln(2)}{\lambda} = 1.8\times10^{11}\ \text{s} = \boxed{5720\ \text{yr}}$$

(b) Initial decay rate $$R_0 = \frac{N_0}{\tau} = N_0\lambda = 3.0\times10^{23}\times10^{-12}\times3.84\times10^{-12}$$

$$\boxed{= 1.15\ \text{Bq}}$$

(c) Decrease in decay rate obeys $$R = R_0\exp(-\lambda t) \;\Rightarrow\; t = \tau\ln\left(\frac{R_0}{R}\right)$$

$$\boxed{t = 4720\ \text{yr}}$$