PA1140 – Waves and Quanta

Unit 4 – Workshop - Solutions

Q 4.1

(a) *Singly ionized* He *behaves like hydrogen with* $Z = 2$ *leading to a factor 4 increase in numerator of the Rydberg-Ritz formula*

$$f = \frac{mk^2Z^2e^4}{4\pi\hbar^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

By inspection, doubling n_f *and* n_i, *will increase denominator by factor 4, thus leading to transitions for* He+ *between even* n *levels that are the same as as those for* H *between levels* $n/2$.

(b) *Writing the formula for wavelength*

$$\lambda = \frac{hc}{E_i - E_f}$$

where $E_n = -Z^2\frac{E_0}{n^2}$, $E_0 = 13.6 \text{ eV}$, $hc = 1240 \text{ eV nm}$

$$= \frac{1240}{-1.51+3.40} = \boxed{656 \text{ nm}}$$

where $E_6 = -1.51 \text{ eV}$, $E_4 = -3.40 \text{ eV}$

Which is the same as the $n = 3$ *to* 2 *transition of* H *(Balmer -alpha)*

Q 4.2 *Assuming the daughter nuclei are initially touching, the electrostatic potential energy is given by*

$$U = k\,\frac{Z_{\mathrm{Ba}} Z_{\mathrm{Kr}} e^2}{R_{\mathrm{Ba}} + R_{\mathrm{Kr}}} \qquad \textit{Where} \quad R_{\mathrm{Ba}} = R_0 A_{\mathrm{Ba}}^{1/3}, \quad R_{\mathrm{Kr}} = R_0 A_{\mathrm{Kr}}^{1/3}$$

$$= \frac{8.99\times10^{9}}{1.2\times10^{-15}} \times \frac{56\times36\times(1.6\times10^{-19})^2}{(144^{1/3}+89^{1/3})} = 3.98\times10^{-11}\ \mathrm{J} = \boxed{249\ \mathrm{MeV}}$$

Q 4.3 *Need to look up half life of ^{14}C (= 5730 yr); so 18000 yr corresponds to*

$n = \dfrac{18000}{5730} \approx 3.141$ *times the half life. Hence the count rate per gram will be:*

$$R = \left(\frac{1}{2}\right)^n R_0 = 1.7\ \text{cts min}^{-1}\ \text{g}^{-1}$$

Thus, to achieve 5 cts/min requires

$$M = \frac{5}{1.7} \approx \boxed{2.94\ \text{g}}$$

Q 4.4

Consider five $^{2}\mathrm{H}$ *which proceed by the two given channels, thus releasing*

$$E = 3.27 + 4.03 + 17.6 = 24.9\ \mathrm{MeV}$$

One mole of water is 18 g, so 4 liters of water (weighing 4 kg) contains

$$N(H) = 2\times\frac{4}{0.018}\times 6.02\times 10^{23} \qquad \textit{since} \quad N_{\mathrm{A}} = 6.02\times 10^{23} \ \textit{atoms/mole}$$

Hence, number of deuterons

$$N(D) = 1.5\times 10^{-4} N(H) \approx 4.01\times 10^{22}$$

And total energy released

$$= 4.01\times 10^{22}\times\frac{24.9\times 10^{6}}{5}\times 1.6\times 10^{-19} = \boxed{3.2\times 10^{10}\ \mathrm{J}}$$