

WAVES AND QUANTA

Unit 3: Introduction to quantum mechanics

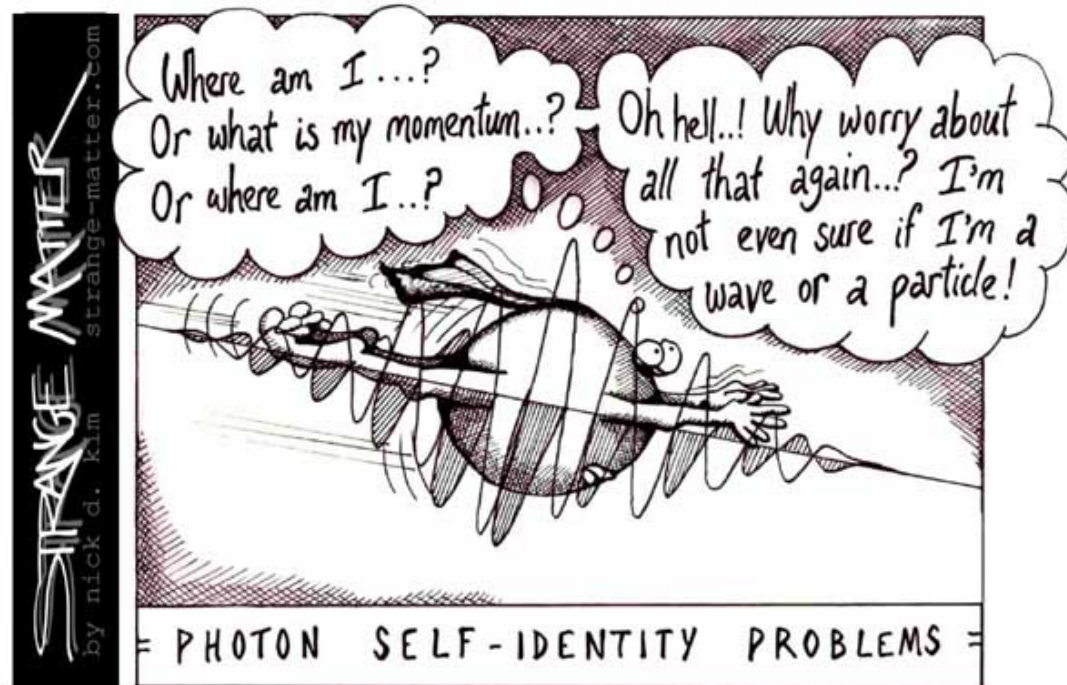
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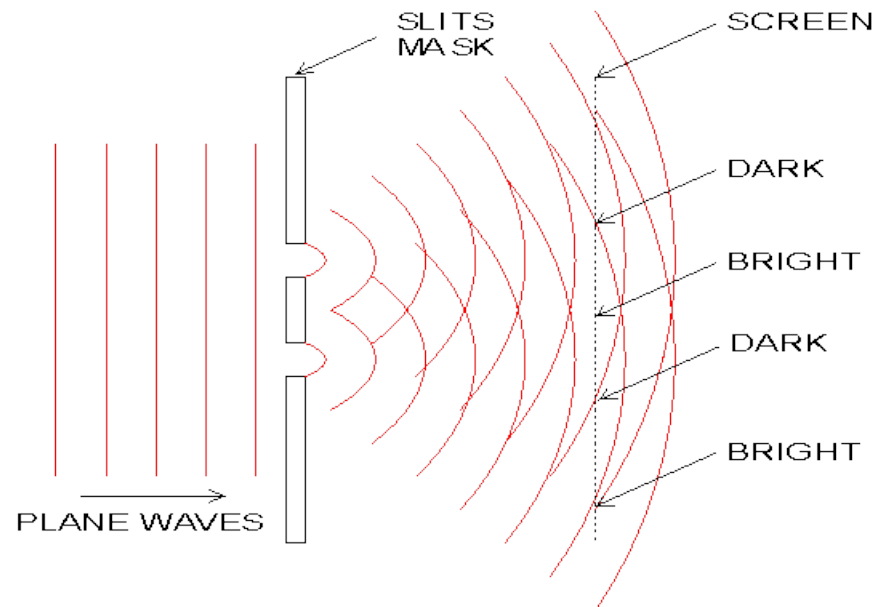
Lecture 1: the basic idea

Reality is fundamentally stranger than was imagined in classical physics. Energy comes in discrete packages and particles obey probabilistic rules and hence behave as if they were waves. In a deep sense, properties of a particle, such as its position and velocity, do not seem to truly exist until they are measured.



Evidence that light is a wave: diffraction and interference

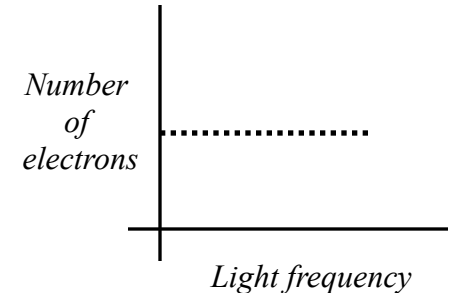
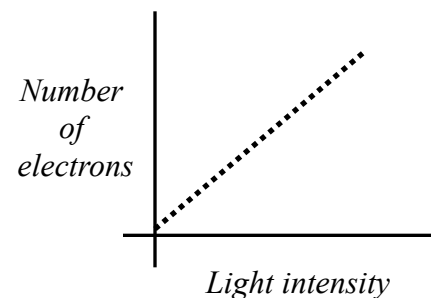
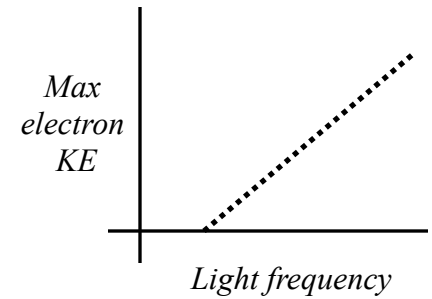
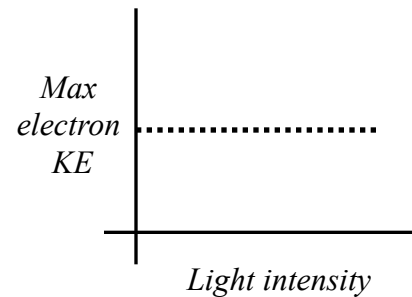
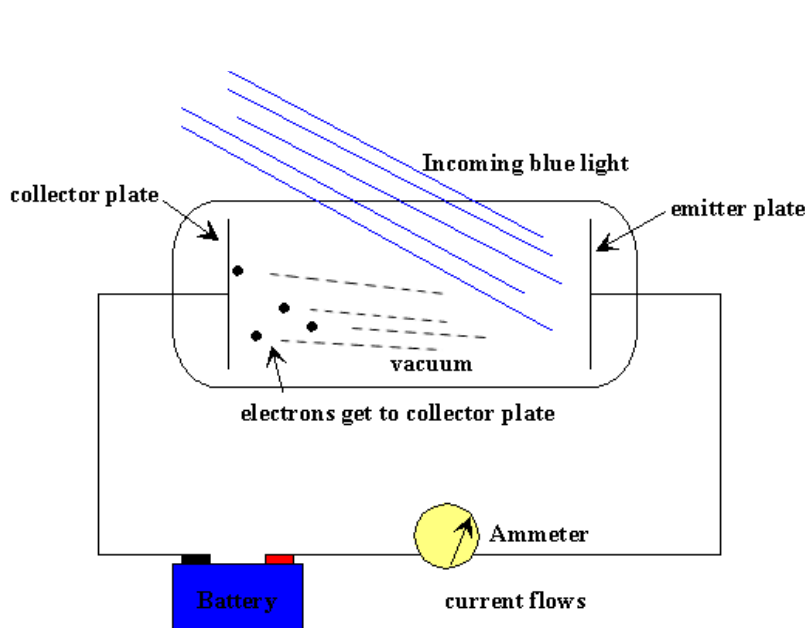
We are familiar with the idea that light (and other forms of **electromagnetic radiation**) is a wave. The evidence is, for example, the constructive and destructive interference seen in **Young's slits** experiment. There is no way of understanding this behaviour in terms of classical particles. Since Clerk-Maxwell we think of these waves as being oscillations in the **electromagnetic field**.



$$\text{Difference in path length} = n\lambda \Rightarrow \text{Constructive interference}$$

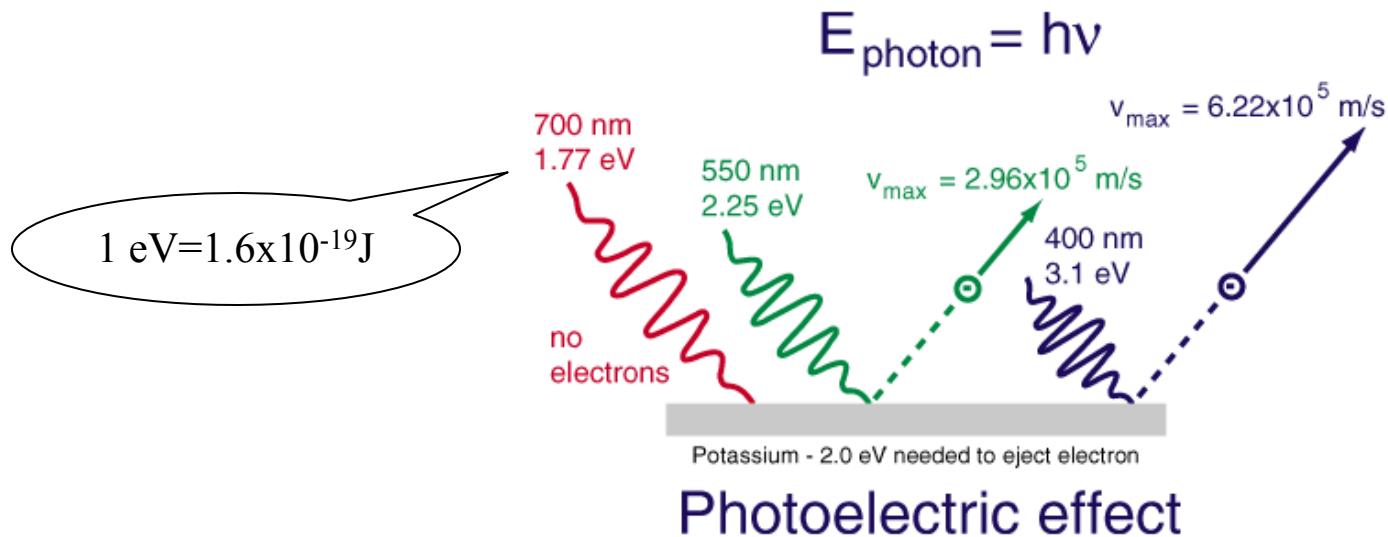
Evidence that light is formed of particles: the photoelectric effect

Newton developed an elaborate and successful theory of light as particles (he called them corpuscles). However, in the decades following his death, this theory gradually became discredited as the evidence for the wave-like behaviour of light built up. By the early 20th century, though, it became clear that light sometimes behaves in ways which can only be understood in terms of localised particles. Most famous of these phenomena is the **photoelectric effect**.



Evidence that light is formed of particles: the photoelectric effect

This behaviour makes no sense in terms of waves, but, as Albert Einstein pointed out in 1905, is perfectly consistent with a particle picture for the light, in which each light particle (**photon**) has an **energy proportional to its frequency**.



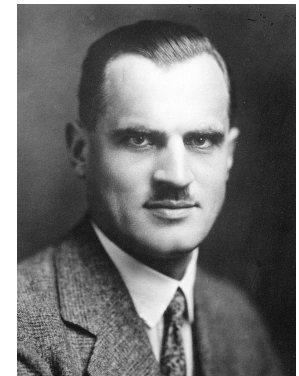
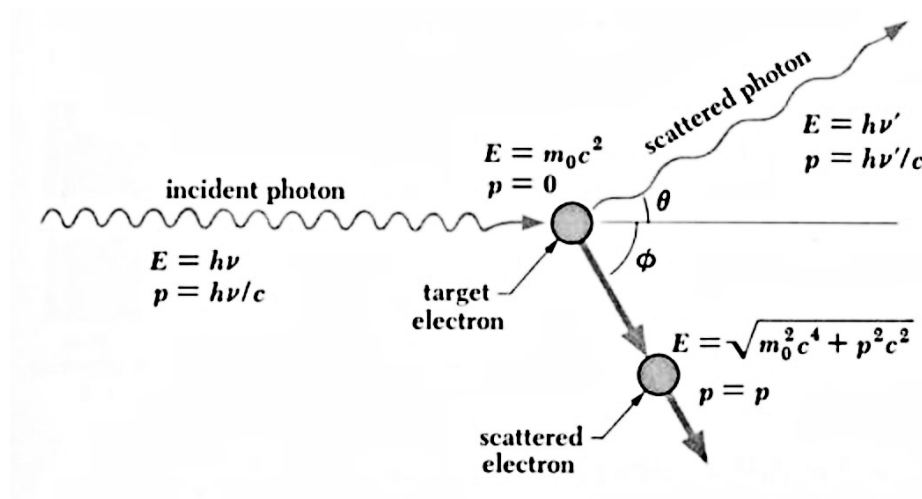
$KE_{\text{max}} = hf - \phi$; where ϕ is the so-called **work function**, which is the minimum energy required to remove an electron, and varies from metal to metal.

N.B. $h = 6.63 \times 10^{-34}$ J s is the **Planck constant**.

Evidence that light is formed of particles:

Compton scattering

Another phenomenon which illustrates the “quantised” particle nature of light is **Compton scattering**. When photons collide with particles such as electrons the collisions obey mechanics of “billiard balls” rather than the more spread out kinds of interactions we would expect of waves.



Momentum before (x direction):

$$p_1$$

Momentum after (resolving in x and y):

$$p_2 \cos \theta + p_e \cos \phi \quad ; \quad p_2 \sin \theta = p_e \sin \phi$$

Energy (relativistic!) before:

$$p_1c + m_e c^2$$

Energy after:

$$p_2c + \sqrt{p_e^2 c^2 + m_e^2 c^4}$$

Evidence that light is formed of particles: Compton scattering

So by conservation of energy and momentum, we conclude (square both momentum equations and combine):

$$p_e^2 = p_1^2 + p_2^2 - 2p_1p_2 \cos \theta$$

Similarly squaring the energy equation gives:

$$p_e^2 = p_1^2 + p_2^2 - 2p_1p_2 + 2m_e c(p_1 - p_2)$$

Combining these we find:

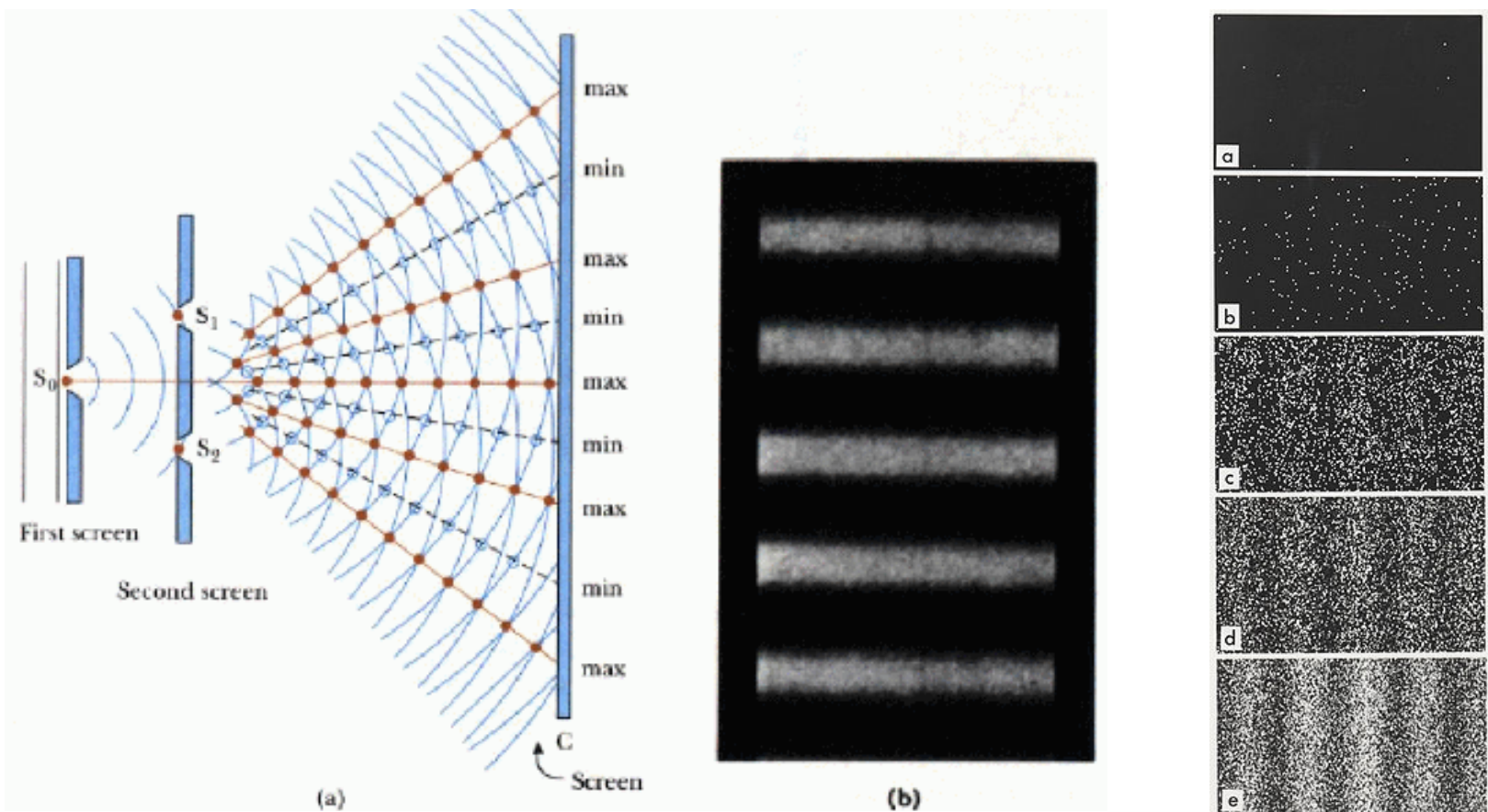
$$p_1p_2(1 - \cos \theta) = m_e c(p_1 - p_2)$$

So the change in wavelength of the photon is given by:

$$\Delta\lambda = \frac{h}{p_2} - \frac{h}{p_1} = h \times \frac{p_1 - p_2}{p_1p_2} = \frac{h}{m_e c} (1 - \cos \theta)$$

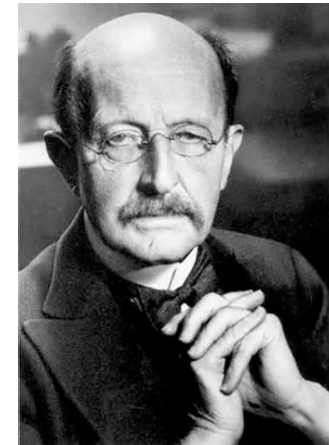
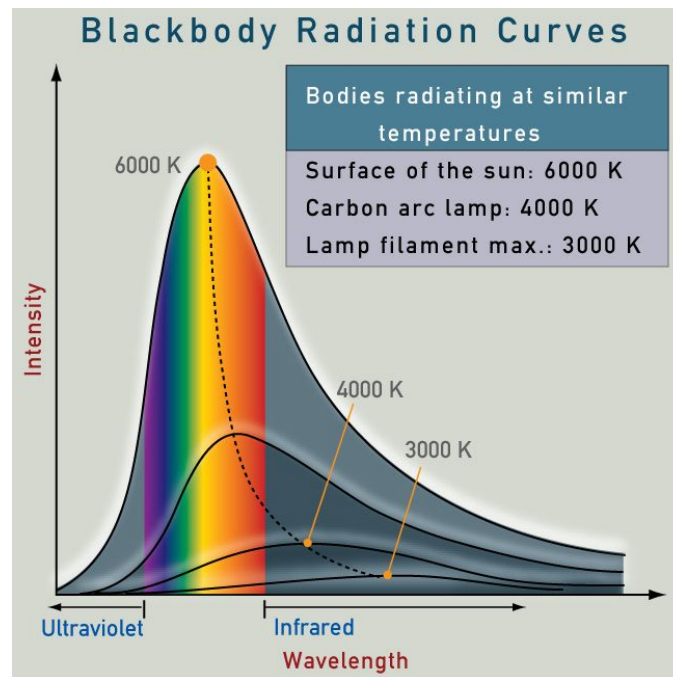
Wave-particle duality

It seems that to understand light we need a theory in which particles have wave-like behaviour in at least some circumstances. This theory is called **quantum mechanics** and it seems to reflect fundamental principles of nature which go beyond classical physics. To appreciate how strange wave-particle duality is, reconsider Young's slits with a detector registering single photons.



Light quanta

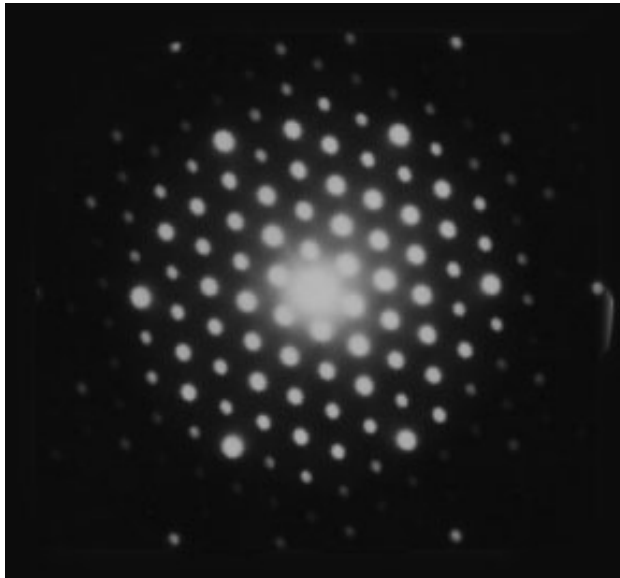
The first suggestion that light might come in discrete lumps came earlier, in 1900, when Max Planck noticed that the shape of a **Black Body spectrum** could be explained if each frequency f came in energy packets of size hf , where $h = 6.63 \times 10^{-34} \text{ J s}$ is the same **Planck constant** we met earlier.



But even though this was quickly seen as a key breakthrough, even Planck was not comfortable with viewing the light quanta as completely real, and instead seems (along with most others) to have seen their introduction as a kind of mathematical trick required to give the right formula.

De Broglie matter waves

In 1924 Prince Louis De Broglie made the radical proposition that not only must light be considered as a wave which behaved in some circumstances like particles, but that traditional particles (electrons, protons, neutrons etc.) behave in some circumstances like waves. This explained phenomena such as electron diffraction through crystals.



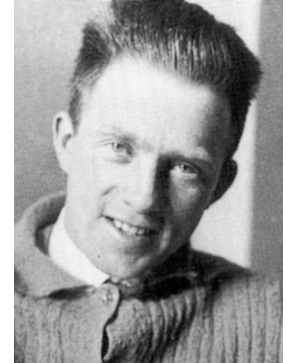
The *de Broglie wavelength* associated with any material particle is simply given by its momentum:

$$\lambda = h/p$$

Heisenberg's Uncertainty Principle

A remarkable consequence of wave-particle duality is that it implies certain pairs of particle properties cannot simultaneously be known to arbitrary accuracy. More quantitatively, considering the uncertainty in position and momentum of a particle, **Heisenberg's Uncertainty Principle** states:

$$\Delta x \Delta p > \hbar/2$$



Where $\hbar = h/2\pi = 1.05 \times 10^{-34}$ J s is the **reduced Planck constant**, or “hbar”.

This restriction can be understood by considering that any measurement of a particle property also disturbs the particle, since measurements require some kind of interaction, such as firing photons at the particle to identify where it is.

However the principle is even more profound than this explanation hints at, seemingly being a fundamental limitation of nature (for greater insight, read about **Bell's Inequalities**).