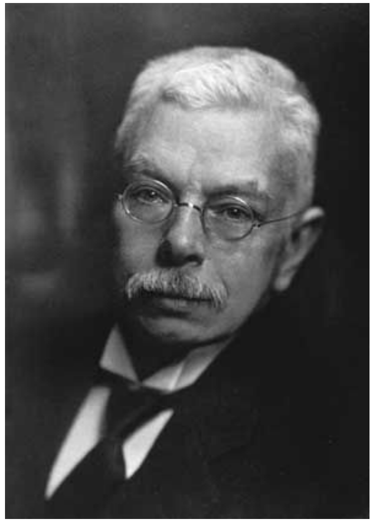
Atoms and Nuclei
PA 322

Lecture 9

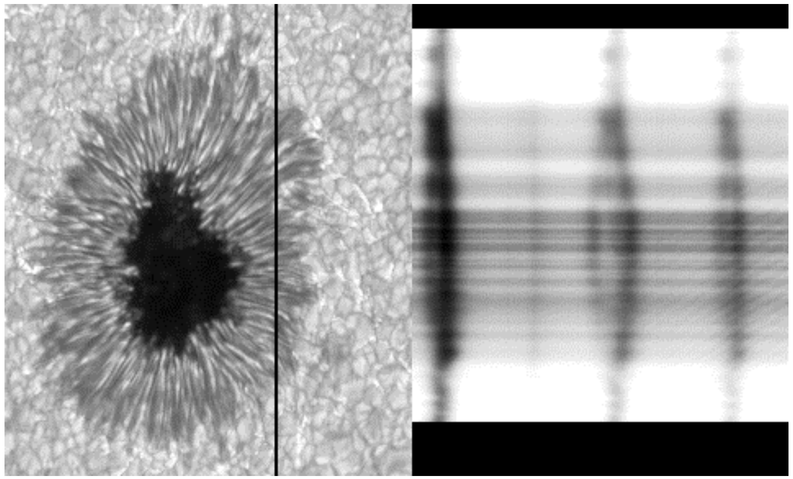
Unit 2:
Zeeman Effect (revisited)
Paschen-Back Effect

(Reminder: http://www.star.le.ac.uk/~nrt3/322)

Topics



- Atom in magnetic field
  - Zeeman Effect: weak field
    - “normal” and “anomalous”
    - magnitude of effect
  - other cases
    - intermediate field: Paschen-Back Effect
    - very strong field



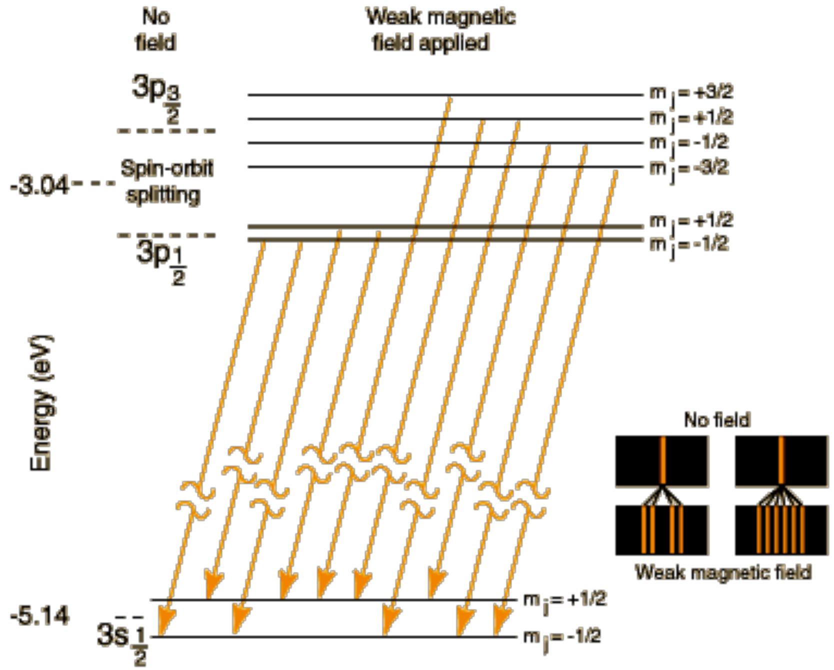
Zeeman Effect

- Atom interacts with external magnetic field as there are intrinsic (internal) magnetic moments associated with:
  - electron orbit and electron spin
- Produces energy splitting of levels that are otherwise degenerate: m_J states
  - and thus splitting of spectral lines to/from relevant levels
  - in typical laboratory magnetic fields the size of the energy splitting due to the *external field* is small compared with all internal energy splitings

    e.g. $E_{mag} << E_{so} << E_{spin\text{-}spin}$
  - equivalently the effective *internal* magnetic field relating to electron magnetic moment interaction >> external field values
  - amplitude of splitting $\propto |\boldsymbol{B}|$

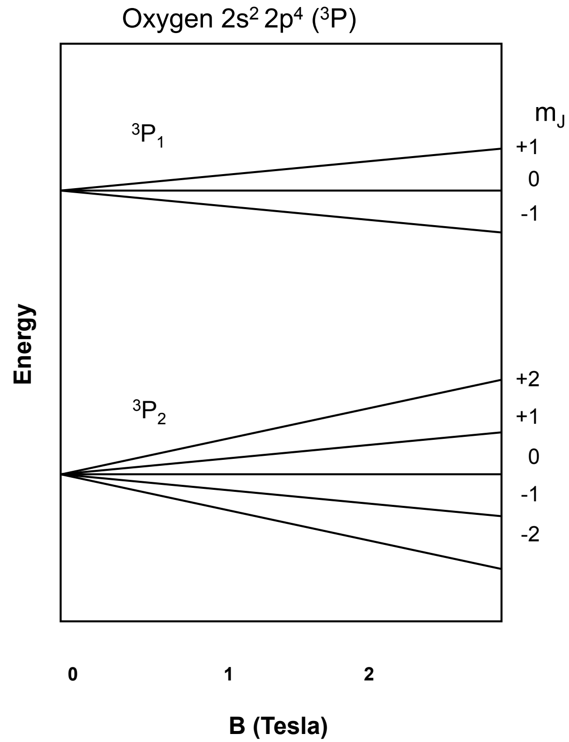
Zeeman Effect

- *Normal Zeeman effect*: splitting of singlet states (S=0) – can be understood classically
- *Anomalous Zeeman effect*: splitting of $S \neq 0$ states, specifically where *spin-orbit coupling* is occurring
  - normal Zeeman effect is thus just a special case, in fact.
  - However, when *Normal Zeeman effect* applies, the line patterns are much simpler.



- *Normal*: number of Zeeman split lines = 3
  - independent of number of transitions
- *Anomalous*: number of Zeeman split lines ≤ number of transitions

Interaction Energy for Zeeman Effect



- Interaction with B field splits levels according to m_{J} (specifying orientations of $\mathbf{J}$)
- Need to calculate is the interaction energy between the effective *magnetic moment* of the atom $\boldsymbol{\mu}$ and the *external field* $\boldsymbol{B}$
- Interaction energy E_{mag} is given by

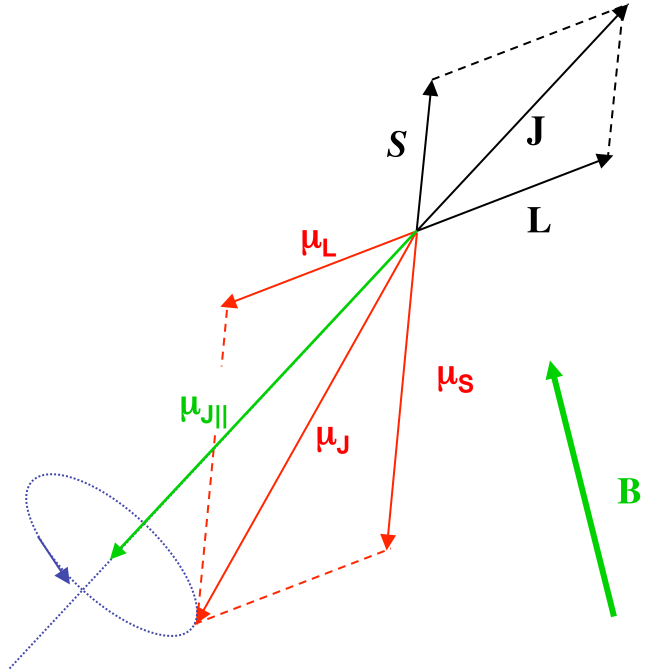
$$E_{\mathrm{mag}} = -\boldsymbol{\mu}\cdot\mathbf{B}$$

where $\boldsymbol{\mu} = \boldsymbol{\mu}_{\mathbf{J}_{\parallel}}$ is the component of $\boldsymbol{\mu}$ which is *antiparallel* to $\mathbf{J}$
(surprisingly μ not in same direction as **-J**)

- The only tricky bit is calculating $\boldsymbol{\mu}_{\mathbf{J}_{\parallel}}$

Interaction Energy for Zeeman Effect

- Assume L-S coupling in multi-electron atom
  - $\mathbf{J} = \mathbf{L} + \mathbf{S}$ ($\mathbf{L}$ & $\mathbf{S}$ precess about $\mathbf{J}$)



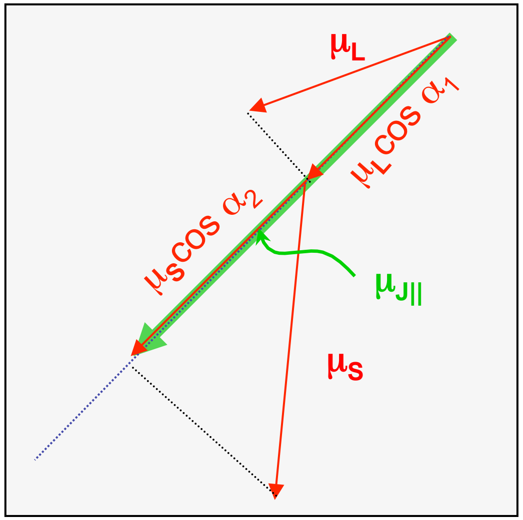
- Orbital magnetic moment: $\boldsymbol{\mu}_{\mathbf{L}} = -g_l\left(\frac{e}{2m_e}\right)\mathbf{L}$
- Spin magnetic moment: $\boldsymbol{\mu}_{\mathbf{S}} = -g_s\left(\frac{e}{2m_e}\right)\mathbf{S}$

  where g_l and g_s are the orbital and spin g-factors $g_l = 1;\ g_s = 2$

- Total magnetic moment $\boldsymbol{\mu}_{\mathbf{J}} = -\left(\frac{e}{2m_e}\right)\left[g_l\mathbf{L} + g_s\mathbf{S}\right] = -\left(\frac{e}{2m_e}\right)\left[\mathbf{L} + 2\mathbf{S}\right]$

confirming that $\hat{\boldsymbol{\mu}}_{\mathbf{J}}$ & $\hat{\mathbf{J}}$ are *not anti-parallel*: $\boldsymbol{\mu}_{\mathbf{J}}$ precesses about $\mathbf{J}$

Interaction Energy for Zeeman Effect



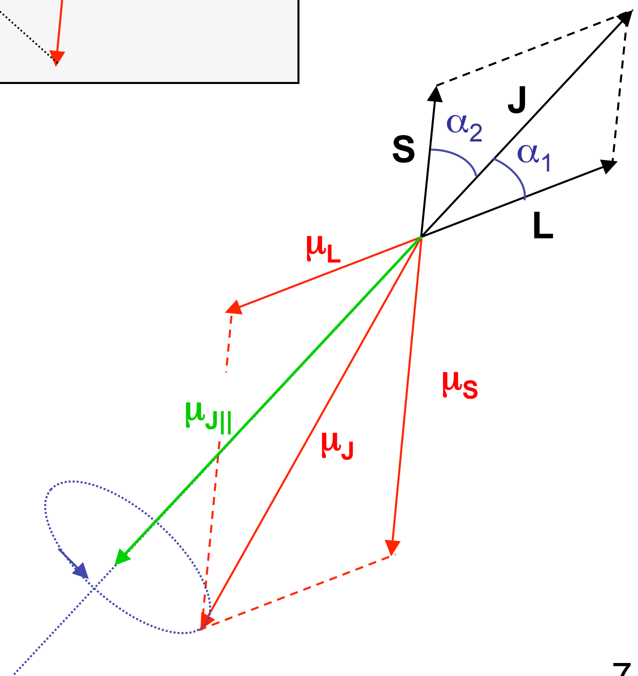
- Computing $\mu_{J_{\parallel}}$

$$\left|\mu_{\mathbf{J}_{\parallel}}\right| = \left|\mu_{\mathbf{L}}\right| \cos\alpha_1 + \left|\mu_{\mathbf{S}}\right| \cos\alpha_2$$

where angles defined in triangles

$$\cos\alpha_1 = \frac{|\mathbf{L}|^2 + |\mathbf{J}|^2 - |\mathbf{S}|^2}{2|\mathbf{J}||\mathbf{L}|} \quad \cos\alpha_2 = \frac{|\mathbf{J}|^2 + |\mathbf{S}|^2 - |\mathbf{L}|^2}{2|\mathbf{J}||\mathbf{S}|}$$

(Cosine rule)



Interaction Energy for Zeeman Effect

So substituting

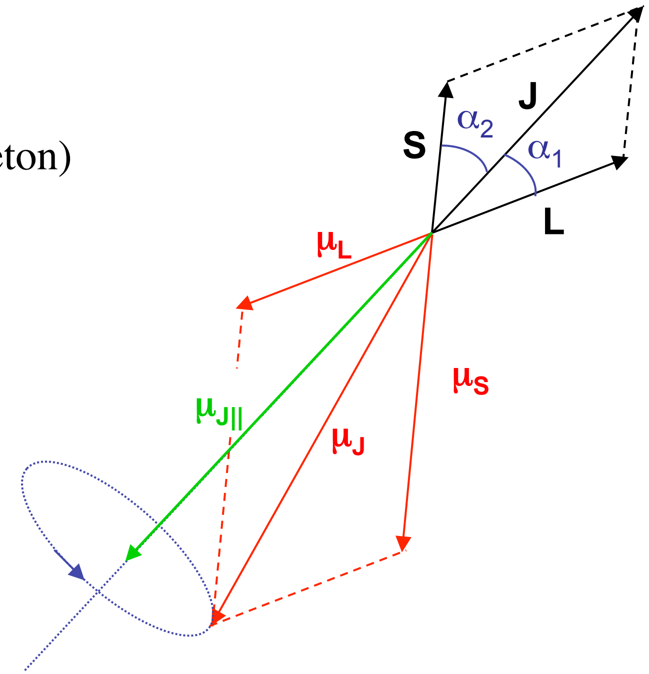
$$|\boldsymbol{\mu}_\mathbf{L}| = -\left(\tfrac{e}{2m_e}\right)|\mathbf{L}| \qquad |\boldsymbol{\mu}_\mathbf{S}| = -2\left(\tfrac{e}{2m_e}\right)|\mathbf{S}| \qquad \mu_B = \frac{e\hbar}{2m_e} \quad \text{(Bohr Magneton)}$$

we get

$$|\boldsymbol{\mu}_{\mathbf{J}_{\parallel}}| = -\frac{\mu_B}{\hbar}\left\{|\mathbf{L}|\left[\frac{|\mathbf{L}|^2+|\mathbf{J}|^2-|\mathbf{S}|^2}{2|\mathbf{J}||\mathbf{L}|}\right]+2|\mathbf{S}|\left[\frac{|\mathbf{J}|^2+|\mathbf{S}|^2-|\mathbf{L}|^2}{2|\mathbf{J}||\mathbf{S}|}\right]\right\}$$

$$|\boldsymbol{\mu}_{\mathbf{J}_{\parallel}}| = -\frac{\mu_B}{\hbar}\left[\frac{3|\mathbf{J}|^2+|\mathbf{S}|^2-|\mathbf{L}|^2}{2|\mathbf{J}|}\right] = -\frac{\mu_B}{\hbar}|\mathbf{J}|\left[1+\frac{|\mathbf{J}|^2+|\mathbf{S}|^2-|\mathbf{L}|^2}{2|\mathbf{J}|^2}\right]$$

$$|\boldsymbol{\mu}_{\mathbf{J}_{\parallel}}| = -\frac{\mu_B}{\hbar}|\mathbf{J}|\boxed{\left[1+\frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)}\right]} = -g\frac{\mu_B}{\hbar}|\mathbf{J}|$$



Expression in brackets is the Landé g-factor

see Softley: p73

$$g = 1+\frac{J(J+1)+S(S+1)-L(L+1)}{2J(J+1)}$$

since $|\mathbf{J}|^2 = J(J+1)\hbar^2$ etc.

Interaction Energy for Zeeman Effect

- Finally

$$E_{mag} = -\boldsymbol{\mu}_{\mathbf{J}_{\parallel}} \cdot \mathbf{B}$$

substitute value of $\boldsymbol{\mu}_{\mathbf{J}_{\parallel}}$

$$E_{mag} = \frac{g\mu_B}{\hbar}\mathbf{J}\cdot \mathbf{B}$$

$$(\text{since } \hat{\boldsymbol{\mu}}_{\mathbf{J}_{\parallel}} = \frac{\boldsymbol{\mu}_{\mathbf{J}_{\parallel}}}{\left|\boldsymbol{\mu}_{\mathbf{J}_{\parallel}}\right|} = -\hat{\mathbf{J}} = \frac{-\mathbf{J}}{|\mathbf{J}|})$$

$$E_{mag} = \frac{g\mu_B}{\hbar}J_z B$$

but $J_z = m_J\hbar$ (projection of J)

$$\Rightarrow \quad E_{mag} = g\mu_B B m_J$$

Interaction Energy for Zeeman Effect

- Limiting cases:
  - $S=0, L \neq 0 \;\Rightarrow\; J=L \;\Rightarrow\; g=1$ (cf. g_l)
  - $S \neq 0, L=0 \;\Rightarrow\; J=S \;\Rightarrow\; g=2$ (cf. g_s)

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

- Normal Zeeman means $S=0$ and thus $g = \text{const} = 1$
  - energy difference (ie line splitting) $\Delta E_{\text{mag}} = \mu_B \, \Delta m_J$ is same for all L [not true for anomalous as $g = g\,(L, S, J)$]
  - simplifies spectrum as different transitions between levels have same energy splitting
  - 3 distinct components only: corresponding to allowed $\Delta m_J = 0, \pm 1$

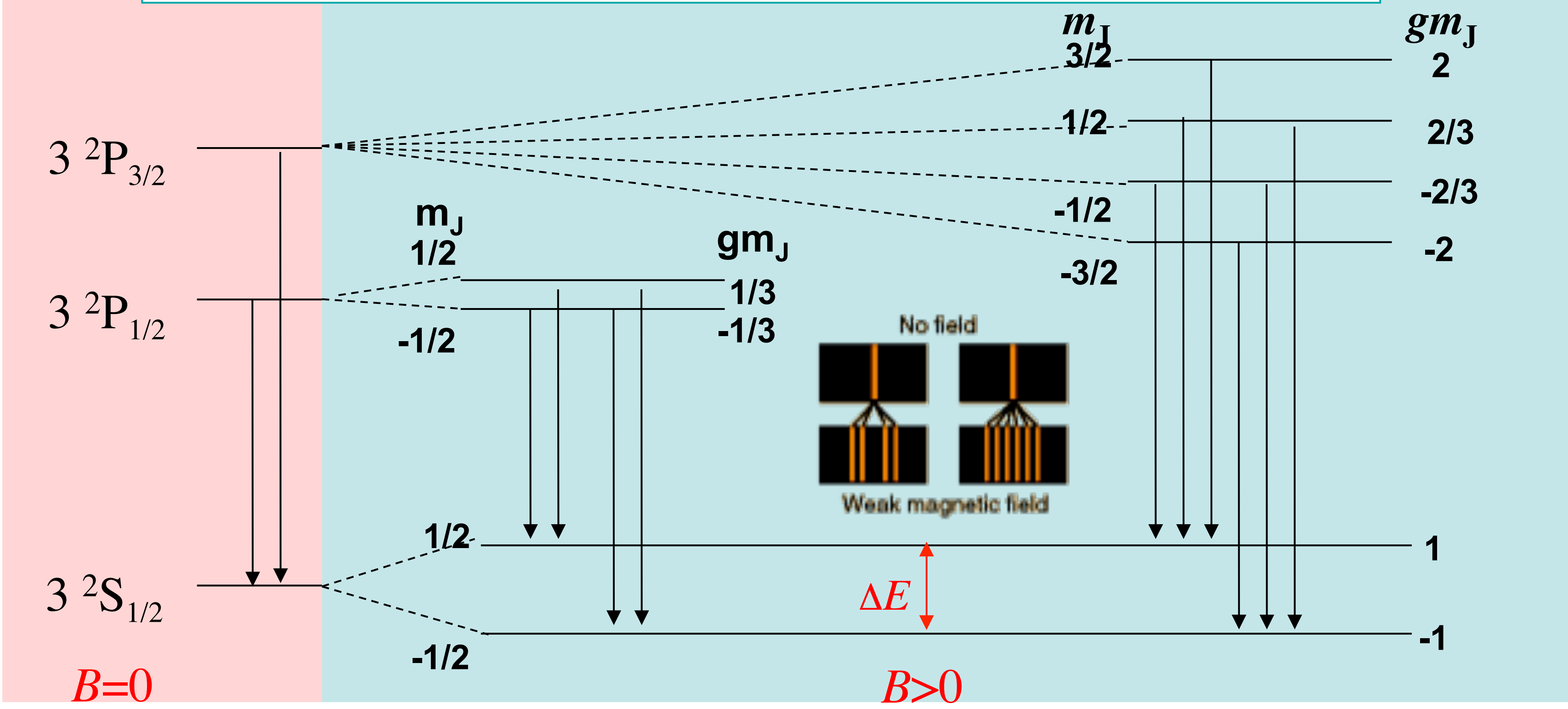
Sodium D lines in a weak (0.1 T) magnetic field

State	L	S	J	g	m_J	gm_J
$3\,^2S_{1/2}$	0	1/2	1/2	2	±1/2	± 1
$3\,^2P_{1/2}$	1	1/2	1/2	2/3	±1/2	±1/3
$3\,^2P_{3/2}$	1	1/2	3/2	4/3	± 1/2; ± 3/2	± 2/3; ± 2

energy **shift** from B=0 case

$$\Delta E \propto \Delta g m_J$$
$$|\Delta\lambda/\lambda| \propto \Delta E/E$$

Selection rules: $\Delta L=\pm1$; $\Delta m_J=0, \pm1$; $\Delta J=0, \pm1$ (not $0\leftrightarrow0$)



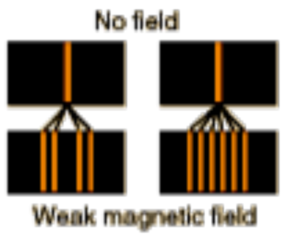
$^2S_{1/2}$ (g=2)		$^2P_{1/2}$ (g=2/3)		Δgm_J $=gm_{J1}-gm_{J2}$
m_J	gm_J	m_J	gm_J	
½	1	½	⅓	4/3
		-½	-⅓	2/3
-½	-1	½	⅓	-2/3
		-½	-⅓	-4/3

$\Delta E \propto \Delta gm_J$

$|\Delta\lambda/\lambda| \propto \Delta E/E$

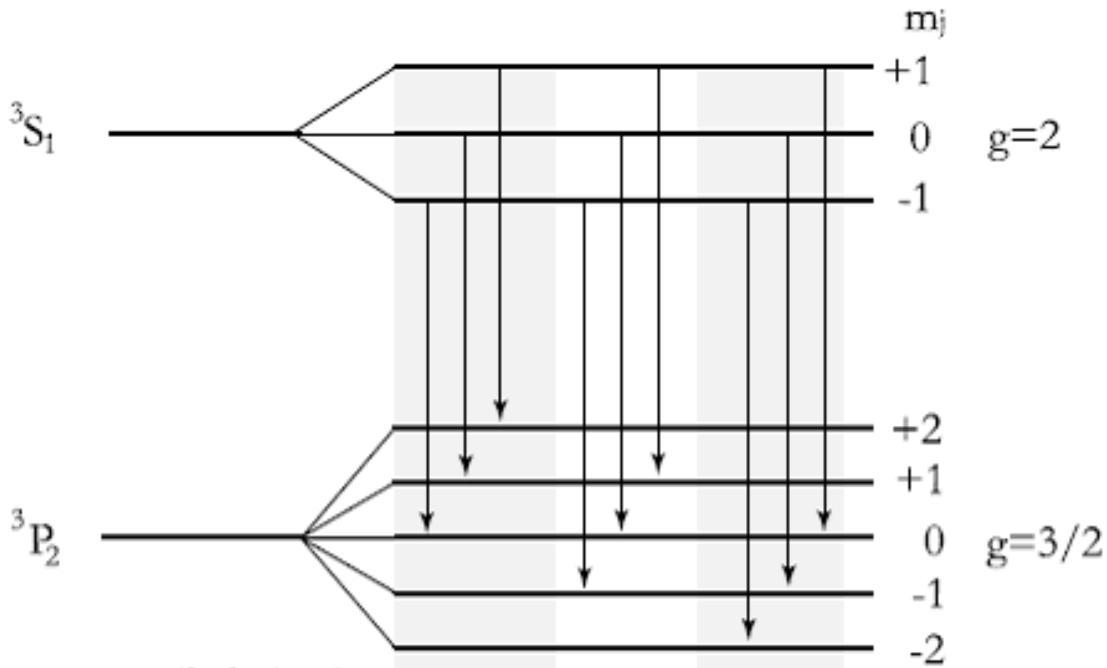
4 distinct components

$^2S_{1/2}$ (g=2)		$^2P_{3/2}$ (g=4/3)		Δgm_J $=gm_{J1}-gm_{J2}$
m_J	gm_J	m_J	gm_J	
1/2	1	3/2	2	-1
		1/2	2/3	1/3
		-1/2	-2/3	5/3
-1/2	-1	1/2	2/3	-5/3
		-1/2	-2/3	-1/3
		-3/2	-2	1



6 distinct components

Another example



S=1 L=0 J=1 ⇒ g = 2

S=1 L=1 J=2 ⇒ g = 3/2

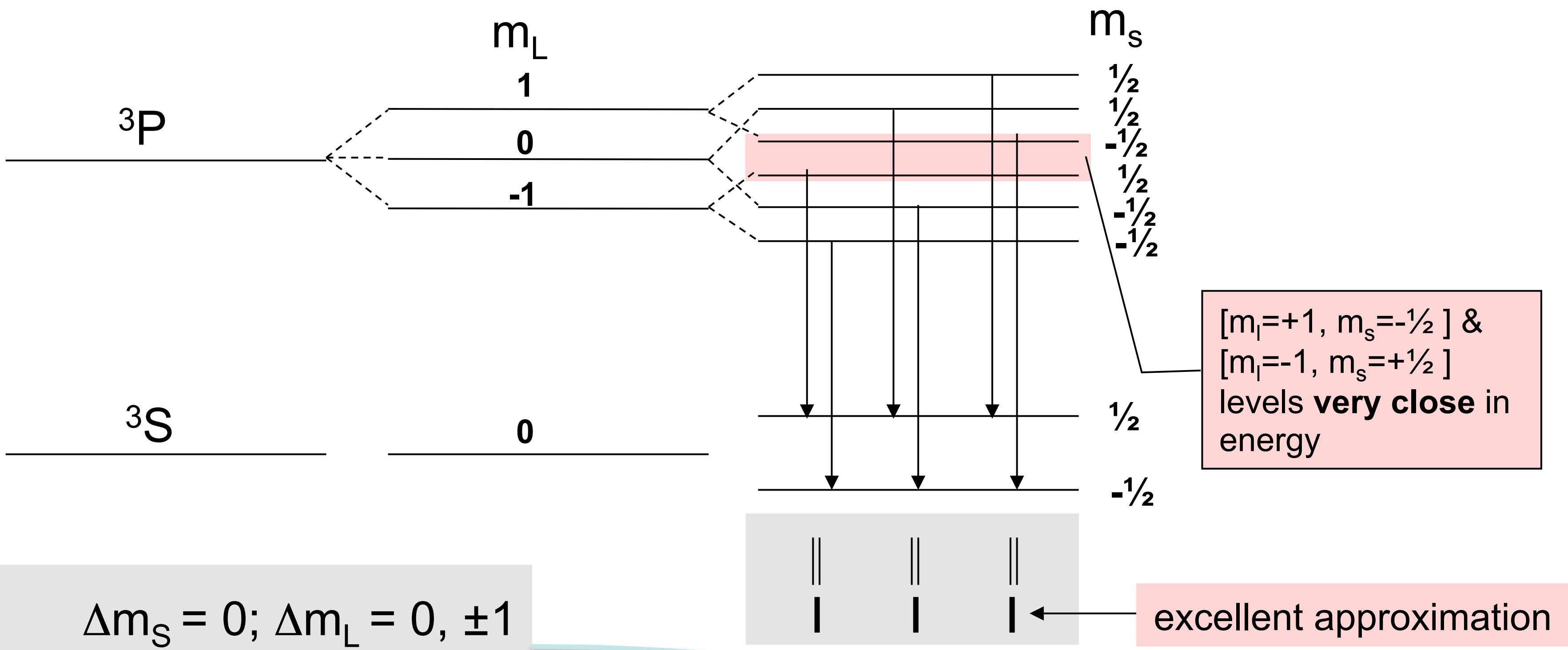
Paschen-Back Effect E_{mag} ~ E_{so}

- Zeeman effect assumes $E_{\mathrm{mag}} << E_{\mathrm{so}}$
- If $E_{\mathrm{mag}} \sim E_{\mathrm{so}} \Rightarrow$ *Paschen-Back Effect*
  - spin-orbit coupling broken
  - $\mathbf{L}$ & $\mathbf{S}$ precess independently about $\mathbf{B}$

$$E_{mag} = -\mu_{\mathbf{L}} \cdot \mathbf{B} - \mu_{\mathbf{S}} \cdot \mathbf{B} = (m_L + 2m_S)\mu_B B$$

  - m_{L} and m_{S} are now good quantum numbers again
  - selection rules for transitions: Δm_{S} = 0; Δm_{L} = 0, ±1
    - produces lines very similar normal to Zeeman effect

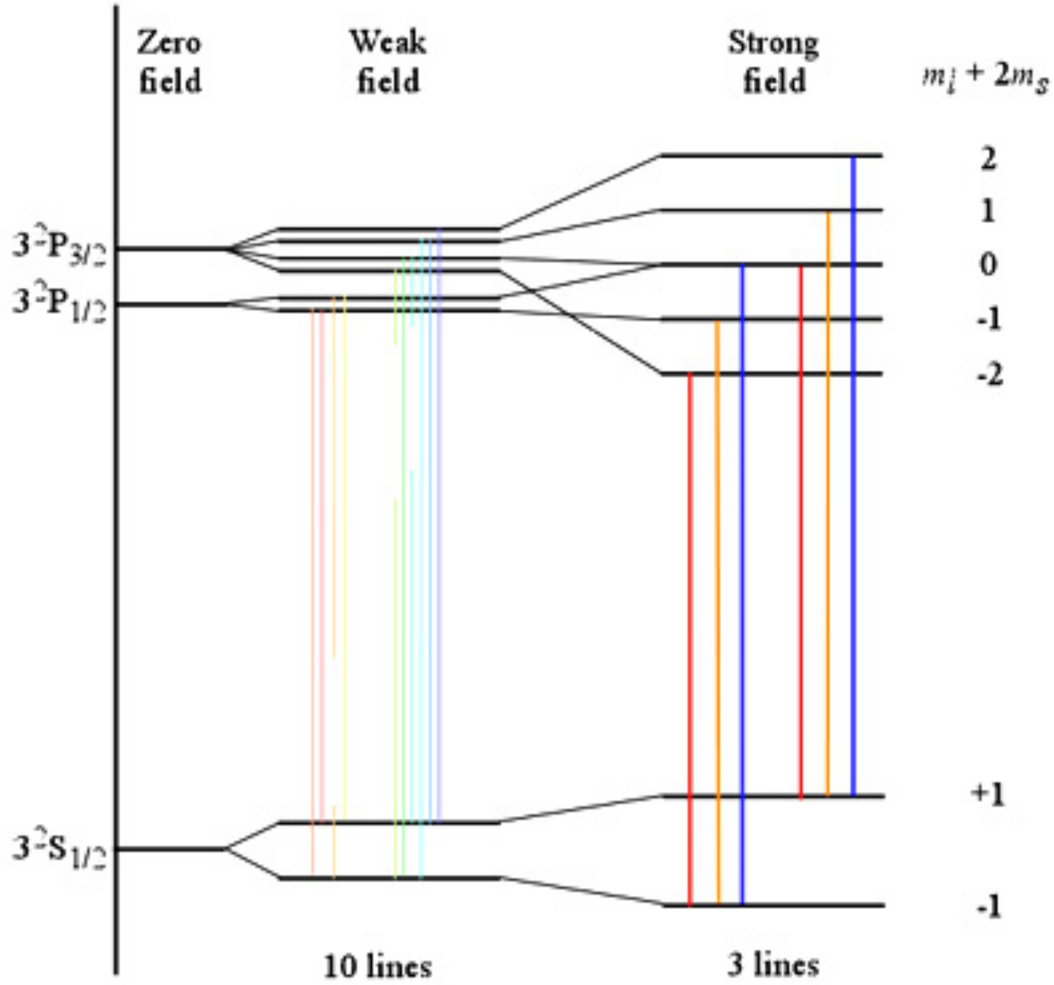
Paschen-Back Effect $E_{mag} \sim E_{so}$



$\Delta m_S = 0; \Delta m_L = 0, \pm 1$

State	L	S	m_L	m_S	m_L+2m_S	$\Delta(m_L+2m_S) \propto \Delta E$
$3\ ^2S_{1/2}$	0	1/2	0	-1/2, +1/2	±1	
$3\ ^2P_{1/2}$	1	1/2	-1,0,1	-1/2, +1/2	-2,-1,0,1,2	-1,0,1
$3\ ^2P_{3/2}$	1	1/2	-1,0,1	-1/2, +1/2	-2,-1,0,1,2	

Zeeman Paschen-Back



Paschen-Back Effect

- Estimate of B for Paschen-Back effect to be important

$$\Delta E_{\text{mag,Zeeman}} = \Delta(g m_J)\mu_B B$$

$$\Delta E_{\text{so}} \approx 2\ \text{meV (e.g. for Sodium resonance lines)}$$

$$\Delta E_{\text{mag,Zeeman}} \approx \Delta E_{so}$$

$$\text{as} \quad \Delta(g m_J)_{\max} \approx 3 \quad \text{and} \quad \mu_B = 5.8\times 10^{-5}\ \text{eV T}^{-1}$$

$$\Rightarrow \quad B \approx 10\ \text{T}$$

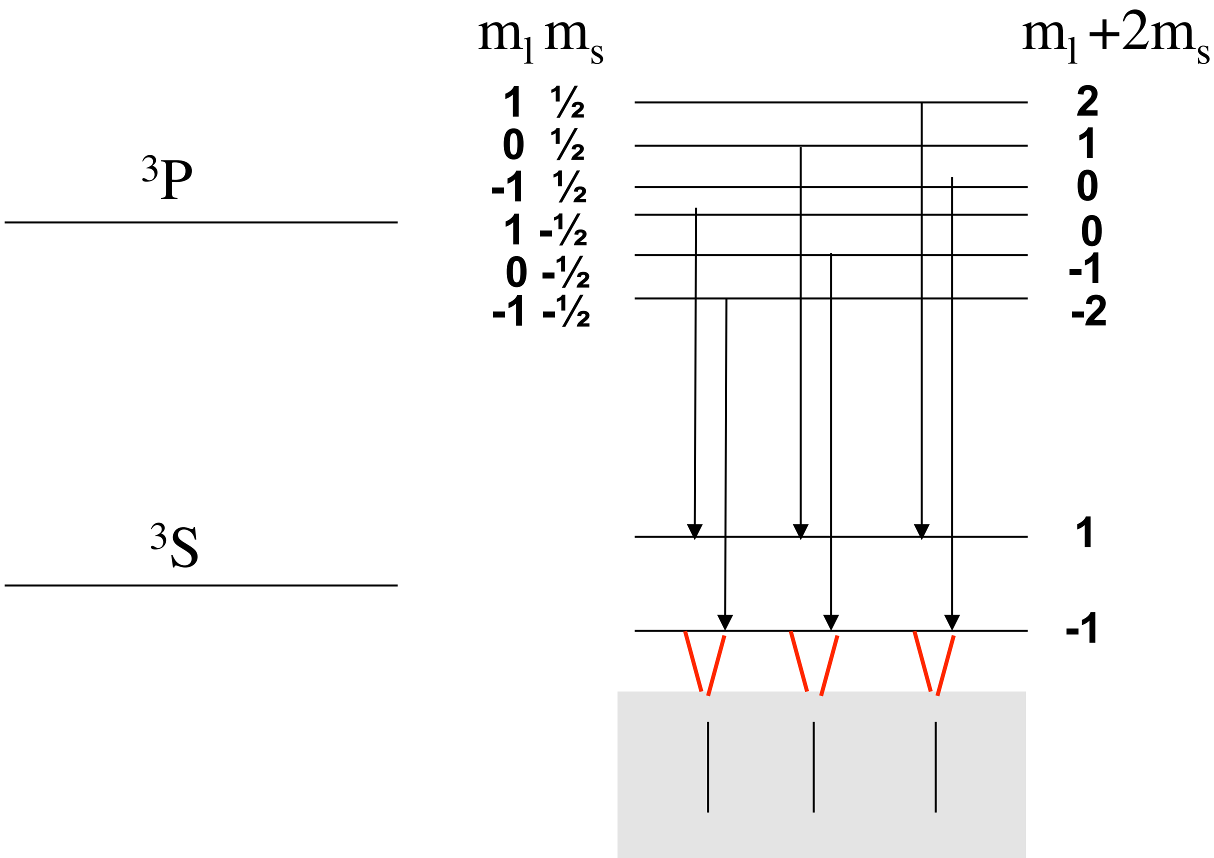
Extreme B fields $E_{mag} >> E_{so}$

- For large enough **B** spin-spin and orbit-orbit coupling of electrons broken, leading to effective *j-j* coupling

$$E_{mag} = (m_L + 2m_S)\mu_B B$$

- Selection rules now $\Delta l = \pm 1$; $\Delta m_J = 0$; $\Delta m_l = 0, \pm 1$

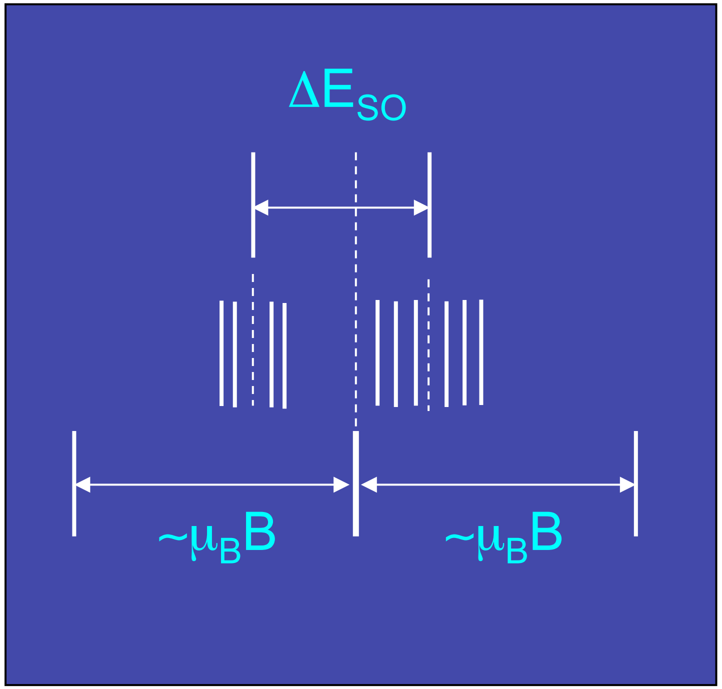
Extreme B fields $E_{mag} >> E_{so}$



Six transitions give three lines due to degeneracy as splitting $\propto (m_l + 2m_s)$

Summary

- Summary of effects for typical lines (e.g. Na D lines):



B=0		doublet
B weak		anomalous Zeeman
B strong		Paschen-Back

Reading

- Previous lecture notes (intro to Zeeman effect)
- Softley Chapter 5, sections 5.4 & 5.5 (*skip Stark effect*)